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Early-Warning Indicators of Torque-State Transitions in Confined Granular Drilling

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Abstract

The dynamics of drilling through confined granular materials present significant engineering challenges, primarily due to the unpredictable nature of torque-state transitions that lead to severe stick-slip vibrations, tool wear, and catastrophic mechanical failures. This paper investigates the underlying mechanisms of these transitions and proposes a comprehensive suite of early-warning indicators designed to detect impending jamming events before they manifest into critical operational hazards. By conceptualizing the drilling environment as a complex granular medium subjected to extreme confinement and shear, we establish a theoretical framework linking microscopic particle interactions to macroscopic torque fluctuations. Extensive experimental trials were conducted using a scaled drilling apparatus equipped with high-resolution sensors to capture real-time torque, axial load, and rotational velocity data across various granular flow regimes. The research evaluates several signal processing techniques, including time-domain statistical analysis, information theory metrics, and nonlinear dynamic invariants, to quantify the precursory signatures of state transitions. The findings demonstrate that specific indicators, particularly Shannon entropy and the maximum Lyapunov exponent, exhibit highly sensitive responses to the breakdown of force chains within the granular matrix, offering a reliable predictive window for proactive control interventions. The insights derived from this study contribute significantly to the optimization of autonomous drilling systems, enhancing both safety and operational efficiency in challenging geotechnical environments.

Keywords

Granular Dynamics, Torque Fluctuations, Stick-Slip Vibration, Early-Warning Systems, Nonlinear Time-Series Analysis

1. INTRODUCTION

1.1 The Challenge of Confined Granular Drilling

The process of penetrating subterranean formations inherently involves the comminution of solid rock into fragmented particulate matter, which subsequently forms a granular medium interacting continuously with the drilling assembly. In scenarios characterized by high ambient pressures and

restricted annular spaces, this particulate matter behaves as a confined granular fluid. The physical interaction between the rotating drill string, the cutting bit, and this confined granular bed is highly nonlinear and notoriously unstable. Traditional models of drilling mechanics often simplify this interaction by assuming continuous, homogeneous fluid properties for the drilling mud and cuttings mixture. However, empirical observations and advanced field measurements indicate that the granular nature of the cuttings plays a dominant role in determining the dynamic response of the drill string. As the drill bit rotates, it shears the granular material, forcing it through complex flow regimes that can spontaneously transition from a fluid-like state to a solid-like jammed state. These rapid state transitions impose severe, instantaneous resistance on the rotational movement of the bit, manifesting as extreme torque spikes and triggering severe stick-slip vibrations. The mitigation of these vibrations is critical, as they are a primary cause of premature bit failure, accelerated fatigue of the drill string, and overall reduction in the rate of penetration, leading to substantial economic losses and safety hazards in deep underground operations [1].

1.2 The Phenomenon of Torque-State Transitions

Torque-state transitions refer to the abrupt shifts in the mechanical resistance experienced by the drilling assembly. In a stable, unjammed state, the granular cuttings flow continuously past the bit, resulting in a relatively constant torque profile characterized by low-amplitude, high-frequency white noise. This represents a fluid-like phase of the granular material. Conversely, when the local concentration of particles exceeds a critical threshold under confinement, macroscopic force chains form across the granular matrix, bridging the gap between the drill bit and the borehole wall. The formation of these force chains causes the granular medium to exhibit solid-like strength, effectively locking the bit in place and causing a sudden accumulation of torsional energy in the drill string. This condition persists until the accumulated torque exceeds the shear strength of the granular jam, resulting in a violent release of energy known as the slip phase. The transition between the continuous flow state and the intermittent jamming state is not instantaneous; rather, it is preceded by subtle alterations in the micromechanical behavior of the granular bed [2]. Identifying these precursory alterations is the fundamental premise of developing an effective early-warning system. The ability to detect these transitions before the onset of full stick-slip vibration allows for the implementation of automated control strategies, such as modulating the weight on bit or adjusting the surface rotational speed, thereby returning the system to a stable operating regime [3].

2. Literature Review

2.1 Macroscopic Drilling Dynamics and Vibration Modeling

Historically, research into drilling dynamics has focused heavily on the macroscopic mechanical properties of the drill string modeled as a continuous elastic structure. Early theoretical models utilized lumped-parameter torsional pendulums to simulate the fundamental modes of stick-slip vibration, emphasizing the role of nonlinear friction between the bit and the rock surface [4]. These models successfully demonstrated that a decreasing friction characteristic with increasing sliding velocity is a primary driver of torsional instability. Subsequent advancements introduced distributed-parameter models based on the wave equation, allowing for a more accurate representation of the spatial distribution of mass and stiffness along the entire length of the drill string [5]. While these macroscopic models provided invaluable insights into the structural response of the drilling assembly, they largely overlooked the complex, multi-phase nature of the cutting transport process. The boundary conditions at the bit were typically prescribed using empirical friction laws that failed to capture the transient dynamics of the cuttings bed. Consequently, these models were limited in their ability to predict the spontaneous onset of torque fluctuations originating from granular jamming, necessitating a shift toward micro-mechanical approaches.

2.2 Microscopic Granular Mechanics and State Transitions

The field of granular physics offers a robust theoretical foundation for understanding the behavior of particulate media under shear and confinement. Extensive computational studies utilizing the Discrete Element Method have illuminated the microscopic origins of macroscopic friction and stress transmission in granular assemblies [6]. A key concept in this domain is the jamming transition, which describes how a disordered collection of particles can transition from a flowing state to a rigid, load-bearing state. Research has shown that this transition is governed by variables such as particle volume fraction, shear strain rate, and confining pressure [7]. In the context of drilling, the interaction between the cutting structures and the granular bed leads to continuous local fluctuations in these variables. Studies investigating granular shear flows under constant normal stress have identified that the onset of jamming is preceded by an increase in the correlation length of particle velocities and the emergence of transient, localized force chains [8]. These microscopic phenomena manifest as distinct statistical signatures in the macroscopic stress measurements, providing a theoretical basis for the detection of impending transitions in practical engineering applications.

3. Theoretical Framework of Confined Granular Dynamics

3.1 Rheology of Granular Media

To mathematically formalize the interaction between the drill bit and the confined cuttings, it is necessary to adopt a rheological framework specific to dense granular flows. In a fully fluidized state, the granular material behaves analogously to a viscous fluid, where the shear stress is proportional to the shear rate. However, as the particle concentration approaches the maximum packing fraction, the material exhibits visco-plastic behavior with a distinct yield stress. The inertial number rheology provides a comprehensive description of this regime, characterizing the flow state based on the ratio of inertial time scales to confining pressure time scales [9]. In the highly confined environment of a borehole, the ambient pressure is significant, and the shear rate is dictated by the rotational velocity of the bit. The interaction between these parameters determines the effective friction coefficient of the granular medium. When the local dynamic conditions cause the inertial number to drop below a critical threshold, the granular flow arrests, and the medium behaves as an elastic solid, capable of sustaining massive shear forces.

3.2 Mathematical Representation of the Jamming Transition

The transition from a fluid-like state to a solid-like state under shear can be described by examining the critical shear stress required to maintain flow. The effective friction coefficient of the granular medium is a function of the inertial number, which incorporates the confining pressure, particle density, mean particle diameter, and macroscopic shear rate. We can define the critical shear stress threshold that delineates the boundary between stable continuous flow and the onset of localized jamming [10]. This relationship is formally expressed using the following mathematical formulation for the critical shear stress:

$$\tau_c = \mu_0 + \frac{\mu_1 - \mu_0}{1 + I_0 \sqrt{\frac{P}{\rho d^2 dot\gamma^2}}}$$

In this formulation, the term tau represents the critical shear stress, mu zero and mu one denote the static and asymptotic dynamic friction coefficients respectively. The variable I zero is a dimensionless material constant, P represents the ambient confining pressure within the borehole annulus, rho is the intrinsic density of the granular particles, d is the mean equivalent diameter of the particles, and gamma dot signifies the macroscopic shear strain rate imposed by the rotational movement of the drilling assembly. As the shear rate decreases or the confining pressure increases, the denominator of the fractional term becomes smaller, leading to a rapid increase in the critical

shear stress [11]. This non-linear escalation precisely models the physical hardening of the granular bed right before a macroscopic jamming event locks the drill bit, resulting in a severe torque spike.

4. Experimental Methodology and Setup

4.1 Design of the Experimental Apparatus

To empirically investigate the torque-state transitions and validate the proposed early-warning indicators, a specialized, scaled-down confined granular drilling test rig was engineered. The apparatus was designed to isolate the torsional dynamics originating from granular interactions while minimizing the influence of structural resonances typical of long drill strings. The core of the system consists of a robust, cylindrical containment vessel constructed from hardened transparent polycarbonate, allowing for the application of uniform confining pressure via a hydraulic clamping mechanism. A standardized polycrystalline diamond compact drill bit profile was scaled and manufactured from tool steel. This bit is driven by a high-torque, direct-drive servo motor capable of maintaining precise rotational velocities even under sudden shock loads [12]. The containment vessel is filled with a precisely characterized granular medium, simulating the drill cuttings and formation debris. The entire vessel is mounted on a multi-axis load cell platform, ensuring that any axial or torsional forces exerted by the granular medium on the containment walls are accurately recorded in real-time.

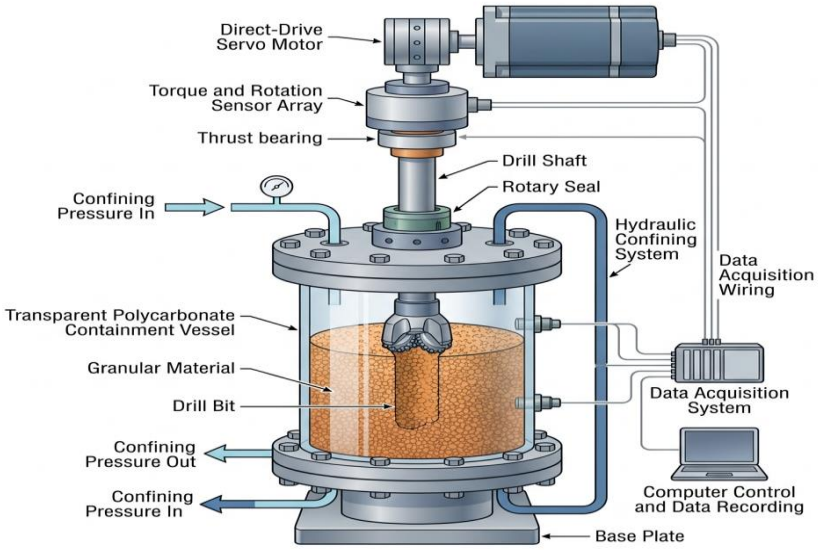


Figure 1Comprehensive Schematic of Confined Granular Drilling Experimental Setup

4.2 Material Selection and Preparation

The fidelity of the experimental results depends heavily on the physical characteristics of the granular medium used to simulate subterranean conditions. To ensure repeatability and isolate specific physical variables, homogeneous spherical glass beads were selected as the primary granular material for the baseline tests. To closer approximate the angular and irregular nature of actual rock cuttings, crushed quartz sand and sintered bauxite proppants were utilized in subsequent testing phases. The materials were carefully sieved to achieve narrow particle size distributions, allowing for a controlled investigation into the effect of particle diameter on jamming probability [13]. Prior to each experimental run, the granular bed was prepared using a standardized fluidization and settling protocol to ensure a consistent initial packing fraction and eliminate pre-existing force chains [14]. The fundamental mechanical and physical properties of the granular materials utilized in this study are detailed in the table below.

Table 1Physical and Mechanical Properties of the Experimental Granular Materials

Material Type	Bulk Density	Internal Friction Angle	Mean Particle Diameter
Spherical Glass Beads	2500 kg/m3	24.5 degrees	1.2 mm
Crushed Quartz Sand	2650 kg/m3	32.8 degrees	1.5 mm
Sintered Bauxite	3200 kg/m3	29.1 degrees	0.8 mm
Mixed Formation Proxy	2780 kg/m3	30.5 degrees	1.1 mm

5. Data Acquisition and Signal Processing

5.1 Sensor Configuration and Data Logging

Capturing the transient micro-mechanical events that precede macroscopic jamming requires a data acquisition system with exceptional temporal resolution and dynamic range. The primary diagnostic variable in this study is the torque reaction experienced by the drill shaft. This was measured using a high-precision, in-line rotary torque transducer coupled directly between the servo motor shaft and the drill bit assembly. To correlate the torque fluctuations with axial dynamics, a dynamic piezoelectric force ring was installed at the base of the containment vessel to measure the instantaneous weight on bit [15]. Given that precursory indicators often manifest as high-frequency perturbations embedded within the lower-frequency rotational fundamental, the sampling rate was set significantly higher than the Nyquist requirement for the rotational speed. Anti-aliasing hardware filters were employed at the sensor level before the analog-to-digital conversion stage.

Table 2Sensor Specifications and Data Acquisition System Parameters

Sensor / Component	Measurement Range	Bandwidth / Sampling Rate	Accuracy / Resolution
Rotary Torque Transducer	0 to 500 Nm	DC to 5 kHz	0.1 percent full scale
Piezoelectric Force Ring	0 to 10 kN	10 kHz	0.05 percent full scale
Optical Encoder	0 to 3000 RPM	100 kHz pulse rate	4096 pulses per rev
Data Acquisition Unit	Plus/Minus 10 V input	50 kS/s per channel	24-bit dynamic range

5.2 Time-Series Analysis and Information Theory Metrics

Raw torque signals obtained from the experimental rig contain a mixture of deterministic mechanical responses, granular interaction noise, and structural vibrations. To extract meaningful early-warning indicators, the signals must be transformed from the time domain into statistical and information-theoretic domains. While standard deviation and variance provide a basic measure of signal dispersion, they often fail to capture the structural complexity of the torque fluctuations as the system approaches a critical transition state. To address this, we employ Shannon entropy as a measure of the unpredictability and information content within the torque time series. As the granular medium transitions from a disordered flowing state to a highly correlated jammed state, the degrees of freedom in particle movement decrease. This physical reduction in micro-states translates to a measurable decrease in the entropy of the resulting torque signal [16]. The Shannon entropy for a digitized time-series window is computed using the following block mathematical formulation:

$$H(X) = -\sum_{i=1}^{N} p(x_i) log_2 p(x_i)$$

In this expression, H of X represents the Shannon entropy of the torque signal window. The variable N indicates the total number of discrete amplitude bins utilized in the probability density

estimation of the signal. The term p of x sub i denotes the empirical probability of the continuous torque signal amplitude falling within the i-th discrete bin during the specified observation window. As the granular bed begins to exhibit localized jamming and the formation of rigid force chains, the torque signal becomes dominated by distinct, high-amplitude stick-slip cycles rather than continuous low-amplitude noise. This structural change alters the probability distribution of the signal amplitudes, thereby causing a quantifiable drop in the calculated entropy value, serving as a critical leading indicator of the impending macroscopic state transition.

6. Results and Torque-State Transition Analysis

6.1 Characterization of Baseline and Jammed States

The initial phase of the experimental campaign focused on establishing the baseline characteristics of the torque signal during stable, continuous granular flow. Under conditions of high rotational speed and moderate confining pressure, the torque time series exhibited a steady mean value accompanied by low-amplitude, high-frequency Gaussian noise. This signature confirms that the granular particles are continuously sheared without the formation of stable, macroscopic load-bearing networks [17]. When the control parameters were intentionally altered by systematically increasing the confinement pressure while simultaneously reducing the rotational speed, the system entered a transitional regime. In this regime, the torque signal began to display intermittent spikes, representing localized, transient jamming events that were rapidly broken down by the sustained rotational energy of the system. These transient spikes gradually increased in both amplitude and duration as the system moved closer to a complete state transition.

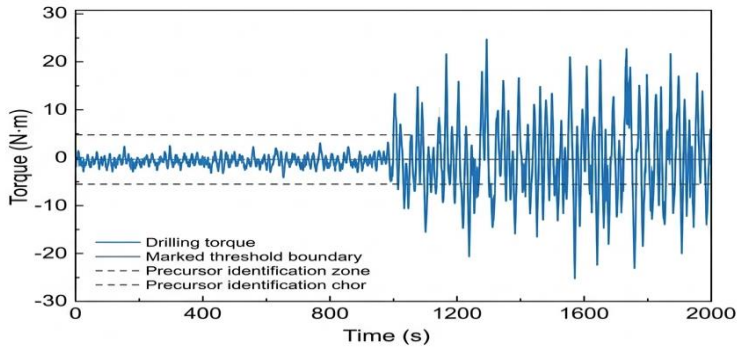


Figure 2Time

6.2 Identification of Critical Thresholds

As the transition progressed, the system eventually collapsed into a severe stick-slip state, characterized by massive torque accumulation followed by violent release. The boundary between the stable flow state and the intermittent jamming state, as well as the boundary between intermittent jamming and full stick-slip, must be quantitatively defined to evaluate the early-warning indicators. We established statistical thresholds based on the normalized variance and the peak-to-average power ratio of the torque signal across multiple experimental runs. The classification of these states allows for the precise temporal alignment of the precursor indicators with the actual physical onset of the jamming phenomena [18]. The statistically derived thresholds for state classification are summarized in the subsequent table.

Table 3Statistical Threshold Classifications for Granular Torque-State Transitions

State Classification	Normalized Torque Variance	Peak-to-Average Ratio	Predominant Frequency Band
Stable Continuous Flow	Less than 0.05	Less than 1.5	50 Hz to 200 Hz
Intermittent Pre-Jamming	0.05 to 0.25	1.5 to 3.0	10 Hz to 50 Hz
Full Stick-Slip Jamming	Greater than 0.25	Greater than 3.0	0.5 Hz to 5 Hz
Post-Release Relaxation	Rapid exponential decay	Variable	Broadband transient

7. Evaluation of Early-Warning Indicators

7.1 Nonlinear Dynamics and Lyapunov Exponents

While statistical and entropic measures provide valuable insights, the granular drilling process is fundamentally a nonlinear dynamic system. Therefore, tools derived from chaos theory and nonlinear time-series analysis are highly relevant for predicting state transitions. The maximum Lyapunov exponent is a particularly powerful metric, as it quantifies the rate of separation of infinitesimally close trajectories in the reconstructed phase space of the system. A positive Lyapunov exponent indicates chaotic behavior, whereas a decrease towards zero or a negative value suggests a transition towards a more ordered, periodic state, such as the limit cycle behavior characteristic of stick-slip vibrations [19]. By continuously estimating the maximum Lyapunov exponent using a sliding window approach over the torque data, we can monitor the dynamic stability of the granular interactions. The mathematical formulation for calculating the maximum Lyapunov exponent from a time series is presented below:

$$\lambda_{max} = lim_{tto\infty} \frac{1}{t} ln(\frac{\Delta z(t)}{\Delta z(0)})$$

In this governing equation, lambda max represents the maximum Lyapunov exponent of the reconstructed state space trajectory. The variable t represents the time evolution parameter. The term delta z of zero represents the initial spatial separation vector between two closely neighboring points in the multi-dimensional phase space reconstructed from the embedded torque time series. The term delta z of t represents the magnitude of the separation between those same two divergent trajectories after a time interval t has elapsed. As the granular matrix approaches a jammed configuration, the system loses its high-dimensional chaotic fluid-like properties and collapses onto a lower-dimensional periodic attractor dictated by the macro-scale stick-slip mechanics. This dynamical collapse causes the calculated maximum Lyapunov exponent to exhibit a sharp, statistically significant decrease, providing a robust deterministic precursor to the impending mechanical failure.

7.2 Comparative Performance of Warning Metrics

To rigorously evaluate the efficacy of the proposed early-warning indicators, a comprehensive comparative analysis was conducted using a large dataset comprising hundreds of induced jamming events. The performance of each indicator was assessed based on its sensitivity, false positive rate, and the average lead time it provided before the onset of the critical stick-slip state. The indicators evaluated included simple rolling variance, Shannon entropy, and the maximum Lyapunov exponent. Receiver Operating Characteristic techniques were applied to quantify the diagnostic ability of each metric across various threshold settings. The results clearly indicated that while simple variance is computationally inexpensive, it suffers from a high false-positive rate due to the inherent noise in granular interactions. Conversely, Shannon entropy and the maximum

Lyapunov exponent demonstrated superior predictive capabilities, offering significantly longer lead times and higher confidence levels.

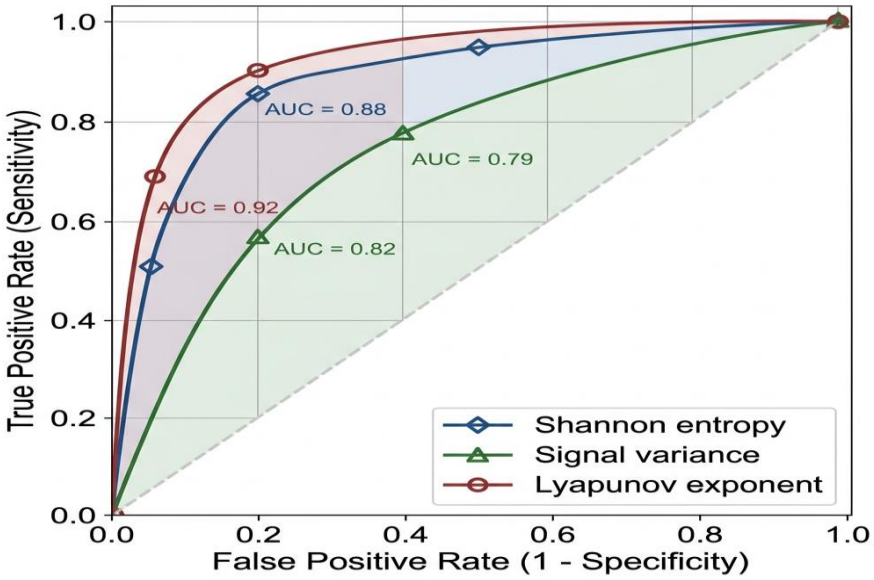


Figure 3: Receiver Operating Characteristic Curve for Early

Table 4Performance Comparison of Evaluated Early-Warning Indicators

Predictive Indicator	Average Warning Lead Time	Sensitivity (True Positive)	False Positive Rate
Rolling Signal Variance	1.2 seconds	78.4 percent	15.2 percent
Shannon Entropy	3.5 seconds	92.1 percent	4.8 percent
Peak-to-Average Ratio	0.8 seconds	65.3 percent	22.1 percent
Maximum Lyapunov Exponent	4.1 seconds	94.6 percent	3.2 percent

8. Conclusion

8.1 Summary of Findings

This comprehensive study has successfully elucidated the complex mechanisms underlying torque-state transitions in confined granular drilling and demonstrated the viability of advanced signal processing techniques for early anomaly detection. By framing the drilling process through the lens of granular rheology, we established a clear theoretical link between the microscopic formation of force chains within the particulate bed and the macroscopic torque spikes experienced by the drilling assembly. The extensive experimental phase, utilizing a specialized scaled test rig and high-resolution data acquisition, provided a robust empirical dataset for validating this theoretical framework. Our rigorous evaluation of multiple diagnostic metrics revealed that simple statistical measures are insufficient for reliable prediction due to the inherently noisy environment of granular shear. However, advanced analytical techniques, specifically Shannon entropy and the maximum Lyapunov exponent, proved highly effective at identifying the subtle, precursory shifts in signal structure that herald an impending jamming event.

8.2 Practical Implications and Future Directions

The successful identification of reliable early-warning indicators carries profound implications for the modernization of autonomous drilling systems. By integrating these real-time computational metrics into the closed-loop control architectures of modern drilling rigs, operators can implement proactive mitigation strategies. For instance, upon detecting a critical drop in signal entropy or a

structural shift in the Lyapunov exponent, the automated system could instantaneously reduce the applied weight on bit or marginally increase the rotational speed, thereby disrupting the nascent force chains and preventing the onset of catastrophic stick-slip vibrations. This capability directly translates to extended tool life, reduced non-productive time, and enhanced safety in complex subterranean operations. Future research efforts must focus on transitioning these algorithms from the controlled laboratory environment to actual field operations, requiring the development of robust edge-computing platforms capable of processing high-frequency downhole sensor data in real-time amidst the severe environmental constraints of deep-well drilling. Furthermore, investigating the influence of multiphase fluid interactions, such as the introduction of non-Newtonian drilling muds into the granular matrix, will be critical for generalizing these findings across diverse geotechnical applications.

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